OPTIMAL OUTSOURCING POLICY FOR A SUPPLY CHAIN MODEL WITH A FINITE REPLENISHMENT RATE, PRICE DEPENDENT DEMAND AND INCREMENTAL HOLDING COST UNDER INFLATION

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Abstract

In this paper, an outsourcing supply chain inventory model is developed for deteriorating items considering price dependent demand. This paper also considers the items deteriorating after a fixed time period called life time and taking incremental holding cost. Effect of inflation has also been taken into account. A comprehensive sensitivity has also been done for some parameters. Cost minimization technique is used to get the approximate expressions for total cost and outsourced quantity.

Keywords: Outsourcing, Supply chain, Price dependent demand, Variable holding cost, Life time items, Inflation.

INTRODUCTION:

Intense competition in today's economy, the shrinking life cycles of products, and the heightening expectations of customers have forced business enterprises to focus their attention on correctly controlling their supply chains.

It is observed all over the world that outsourcing has become one of the most untapped and significant opportunity for gaining sustained competitive advantage for most supply chains. Outsourcing is increasingly considered as a value creation that can provide companies with a long-lasting competitive advantage.

It has been evident for a long time that inventory control and the need for coordination of inventory decisions are important issues in the supply chain. One of the important concerns of the supply chain management is to decide when and how much to outsource or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory undergo decay or deterioration. Various types of inventory models for items deteriorating at a constant rate were discussed by Roychowdhury and Chaudhuri (1983), Padmanabhan and Vrat (1995), Balkhi and Benkherouf (1996) and Yang (2005) etc. Most models that consider the deteriorating items are deteriorate from zero time. But this is not true in realistic, since each item deteriorate after fixed time period called life time. There are few models are developed in which life time has been taken an important factor. Hwang and Hahn (2000), developed an optimal procurement policy for items with an inventory level dependent demand rate and fixed life time.

In the present competitive market, the selling price of an item is one of the decisive factors to the customers. It is commonly seen that lesser selling price causes increase in the demand whereas higher selling price has the reverse effect. Hence, the demand of an item is dependent on the selling price of that item. Burwell et. al. (1997), developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price dependent demand rate was considered by Mondal, et. al (2003). You (2005), developed an inventory model with price and time dependent demand.

In the storage of deteriorating and perishable items, the assumption that the holding cost is constant for the entire inventory cycle is particularly false. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed and therefore, the higher the holding cost.

Giri et al. (1996), developed an EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost are continuous functions of time. Alfares (2007), developed an inventory model with stock dependent demand rate and variable holding cost. In this model, two type time dependent holding cost step functions are considered, retroactive holding cost increase and incremental holding cost increase.

In the past, most work has been done by many authors under consideration negligible inflation. But in recent times many countries have been confronted with fluctuating inflation rates that often have been far from negligible level. The pioneer in this field was Buzancott (1975), who developed the first EOQ model taking inflation into account. Mishra (1979), first provided different inflation rates for various costs associated with an inventory system, under the assumption of constant demand. Bose et al. (1995) developed the EOQ inventory model under inflation and time discounting. Yang et al. (2001), provided an inventory models with time varying demand patterns under inflation. Yang (2004), considered a two-warehouse inventory problem for deteriorating items with constant demand rate under inflation.

This paper discusses a model to determine an outsourcing policy for a supply chain under the influence of different decision criteria such as finite replenishment rate, price dependent demand rate, deterioration rate and time varying holding cost. The purpose of this research is to aid the manufacturer in economically outsourcing the inventory, where the decision is influenced by the time value of money and inflation.

Assumptions and Notations:

The following assumptions are made in developing the present mathematical model of supply chain system:

- The replenishment of the outsourced units starts at a finite rate and stops where the inventory attains its maximum level.
- The demand rate, D(s) is a linear, non-negative, continuous, convex, decreasing function of the selling price, D(s) = a – b(s), where a & b are both positive constants.
- A single item is considered, which deteriorates after a fixed time period called life time. There is no replacement or repair of deteriorating items during the period under consideration.
- Lead time is zero.
- The replenishment rate p is finite and constant, where p > D(s).
- Varying holding cost is considered and it is applied to good units only.
- Shortages are fully backlogged.

The following notations are made in developing the present mathematical models of the supply chain system:

$I_i(t_i)$	Inventory level at any time t_i, $0 \le t_i \le T_i$, i = 1, 2, 3, 4.
$\theta(t)$	Variable rate of deterioration
T_1	The time where the inventory attains its maximum level and replenishment stops.
μ	The life time of items and deterioration of the items is considered only after the life time of items and θ(t) is the variable deterioration rate, s.t. $\theta(t) = \theta t$, $0 < \theta << 1$
I_m	Maximum outsourced inventory level, after fulfilling backorders.
C_1	Set up cost for each replenishment.
C_3	Shortage cost.
h	Holding cost during the life time.
k_m	Holding cost during the period m, where m is the number of distinct time periods with different holding rates after life time period i.e. during the period T_1.
l_n	Holding cost during the period n, where n is the number of distinct time periods with different holding rates during period T_2.
R	The inflation rate.
Q	Outsourced units

MODEL DEVELOPMENT:

In this model, the manufacturer outsources the whole of its production. The replenishment of the outsourced units starts at time T_3 to build up the maximum inventory level Q until time T_1 where the replenishment stops. During the period μ, the inventory level is affected by demand and replenishment. After life time, i.e. during the period T_1 the inventory level changes due to replenishment, demand and deterioration. Due to reasons of demand and deterioration of items, the inventory level gradually diminishes during the period T_2 and ultimately falls to zero at time T_2. T_3 is the period in which inventory level falls below zero and shortage starts to accumulate after which the replenishment is restarted to fulfill the demand and shortage.

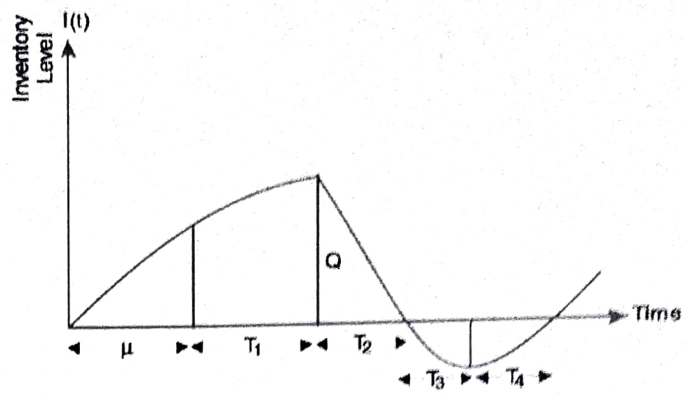

Fig 5.1: Graphical representation of inventory system

The inventory can be described by the following differential equations:

$$\frac{dI_\mu(t_\mu)}{dt_\mu} = p-(a-bs), \qquad 0 \le t_\mu \le \mu \qquad ..(1)$$

$$\frac{dI_1(t_1)}{dt_1} + \theta t_1 I_1(t_1) = p-(a-bs), \qquad 0 \le t_1 \le T_1 \qquad ..(2)$$

$$\frac{dI_2(t_2)}{dt_2} + \theta t_2 I_2(t_2) = -(a-bs), \qquad 0 \le t_2 \le T_2 \qquad ...(3)$$

$$\frac{dI_3(t_3)}{dt_3} = -(a-bs), \qquad 0 \le t_3 \le T_3 \qquad ...(4)$$

$$\frac{dI_4(t_4)}{dt_4} = p-(a-bs), \qquad 0 \le t_4 \le T_4 \qquad ...(5)$$

om equation (1), we have

$$I_\mu(t_\mu) = \{p-(a-bs)\}t_\mu + c_1$$

By applying the boundary condition $I_\mu(0) = 0$, the solution of the equation is given by

$$I_\mu(t_\mu) = \{p-(a-bs)\}t_\mu \qquad 0 \le t_\mu \le \mu \qquad ...(6)$$

From equation (2), we have

$$I_1(t_1)e^{\theta t_1^2/2} = \{p-(a-bs)\}\left(t_1 + \frac{\theta t_1^{3}}{6}\right) + c_2$$

By applying the boundary condition $I_\mu(\mu) = I_1(0)$, the solution of the equation is given by

$$I_1(t_1) = \{p-(a-bs)\}e^{-\theta t_1^2/2}\left(t_1 + \frac{\theta t_1^{3}}{6} + \mu\right) \qquad 0 \le t_1 \le T_1 \qquad ..(7)$$

From the equation (3), we have

$$I_2(t_2)e^{\theta t_2^2/2} = -(a-bs)\left(t_2 + \frac{\theta t_2^3}{6}\right) + c_3$$

By applying the boundary condition, $I_1(T_1) = I_2(0) = Q$, we have

$$I_2(t_2) = -(a-bs)e^{-\theta t_2^2/2}\left(t_2 + \frac{\theta t_2^3}{6}\right) + Qe^{-\theta t_2^2/2} \qquad 0 \le t_2 \le T_2 \qquad ..(8)$$

From equation (4), we have

$$I_3(t_3) = -(a-bs)t_3 + c_4$$

By applying the boundary condition, $I_3(0)=0$, we have

$$I_3(t_3) = -(a-bs)t_3 \qquad 0 \le t_3 \le T_3 \qquad ..(9)$$

From equation (5), we have

$$I_4(t_4)e^{\theta t_4^2/2} = \{p-(a-bs)\}t_4 + c_5$$

By applying the boundary condition, $I_4(T_4) = 0$.The solution of the equation is given by

$$I_4(t_4) = \{p-(a-bs)\}(t_4 - T_4) \qquad 0 \le t_4 \le T_4 \qquad ..(10)$$

Because the maximum inventory takes place at time T_1 , we have

$$I_m = I_1(T_1) = \{p-(a-bs)\}e^{-\theta T_1^2/2}\left(T_1 + \frac{\theta T_1^3}{6} + \mu\right) \qquad ..(11)$$

Therefore from equation (8), we have

$$I_2(t_2) = -(a-bs)e^{-\theta t_2^2/2}\left(t_2 + \frac{\theta t_2^3}{6}\right) + \{p-(a-bs)\}\left(T_1 + \frac{\theta T_1^3}{6} + \mu\right)e^{-\theta(t_2^2+T_1^2)/2} \qquad ...(12)$$

The outsourced quantity for each cycle is given by

$$Q = \int_0^{\mu} D(s)dt_\mu + \int_0^{T_1} D(s)dt_1 + \int_0^{T_2} D(s)dt_2 + \int_0^{T_3} D(s)dt_3 + \int_0^{T_4} D(s)dt_4 + \int_0^{T_1} \theta t_1 I_1(t_1)dt_1 + \int_0^{T_2} \theta t_2 I_2(t_2)dt_2$$

$$= \int_0^{\mu}(a-bs)dt_\mu + \int_0^{T_1}(a-bs)dt_1 + \int_0^{T_2}(a-bs)dt_2 + \int_0^{T_3}(a-bs)dt_3 + \int_0^{T_4}(a-bs)dt_4$$

$$+ \int_0^{T_1}\theta t_1\{p-(a-bs)\}\left(1-\frac{\theta t_1^2}{2}\right)\left(t_1 + \frac{\theta t_1^3}{6} + \mu\right)dt_1 - \int_0^{T_2}\theta t_2(a-bs)\left(1-\frac{\theta t_2^2}{2}\right)$$

$$+\left(t_2 + \frac{\theta t_2^3}{6}\right)\int_0^{T_2}\theta t_2\{p-(a-bs)\}\left(T_1 + \frac{\theta t_1^3}{6} + \mu\right)e^{-\theta(T_1^2+t_2^2)/2}\}dt_2$$

$$= (a-bs)\left(\mu + T_1 + T_2 + T_3 + T_4 - \frac{\theta T_2^3}{3}\right) + \{p-(a-bs)\}\left(\frac{\theta T_1^3}{3} + \frac{\theta \mu T_1^2}{2} + \frac{\theta T_1 T_2^2}{2} + \frac{\theta \mu T_2^2}{2}\right) \qquad ...(13)$$

Since the ordering is made at the beginning of the cycle, the inflation does not affect the ordering cost Therefore,

$$OR = \frac{1}{T} C_3 \qquad \text{...(14)}$$

Holding cost (HD) is given by

$$HD = \frac{1}{T}\Bigg[h\int_0^{\mu} e^{-Rt_\mu} I_\mu(t_\mu)dt_\mu + k_1 \int_0^{\phi_1} e^{-R(\mu+t_1)} I_1(t_1)dt_1 + k_2 \int_{\phi_1}^{\phi_2} e^{-R(\mu+t_1)} I_1(t_1)dt_1 + \dots$$

$$+ k_m \int_{\phi_{m-1}}^{\phi_m = T_1} e^{-R(\mu+t_1)} I_1(t_1)dt_1 + l_1 \int_0^{\psi_1} e^{-R(\mu+T_1+t_2)} I_2(t_2)dt_2 + l_2 \int_{\psi_1}^{\psi_2} e^{-R(\mu+T_1+t_2)} I_2(t_2)dt_2 + \dots$$

$$+ l_n \int_{\psi_{n-1}}^{\psi_n = T_2} e^{-R(\mu+T_1+t_2)} I_2(t_2)dt_2 \Bigg]$$

$$= \frac{1}{T}\Bigg[h\int_0^{\mu} e^{-Rt_\mu} I_\mu(t_\mu)dt_\mu + \sum_{i=1}^{m} k_i \int_{\phi_{i-1}}^{\phi_i(\phi_m = T_1)} e^{-R(\mu+t_1)} I_1(t_1)dt_1 + \sum_{j=1}^{n} l_j \int_{\psi_{j-1}}^{\psi_j(\psi_n = T_2)} e^{-R(\mu+T_1+t_2)} I_2(t_2)dt_2 \Bigg]$$

$$= \frac{1}{T}\Bigg[h\int_0^{\mu} (1 - Rt_\mu)\{p-(a-bs)\}t_\mu dt_\mu + \sum_{i=1}^{m} k_i \int_{\phi_{i-1}}^{\phi_i(\phi_m = T_1)} (1 - R\mu - Rt_1)\{p-(a-bs)\}e^{-\theta t_1^2/2}$$

$$\left(t_1 + \frac{\theta t_1^3}{6} + \mu\right)dt_1 + \sum_{j=1}^{n} l_j \int_{\psi_{j-1}}^{\psi_j(\psi_m = T_2)} (1 - R\mu - RT_1 - Rt_2)\Big\{-(a-bs)e^{-\theta t_2^2/2}\left(t_2 + \frac{\theta t_2^3}{6}\right)$$

$$+ (p-(a-bs))e^{-\theta(t_2^2+T_1^2)/2}\left(T_1 + \frac{\theta T_1^3}{6} + \mu\right)\Big\}dt_2 \Bigg]$$

$$= \frac{1}{T}\Bigg[h\{p-(a-bs)\}\int_0^{\mu} (t_\mu - Rt_\mu^2)dt_\mu + \sum_{i=1}^{m} k_i \int_{\phi_{i-1}}^{\phi_i(\phi_m = T_1)} (1 - R\mu - Rt_1)\{p-(a-bs)\}$$

$$\left(1 - \frac{\theta t_1^2}{2}\right)\left(t_1 + \frac{\theta t_1^3}{6} + \mu\right)dt_1 + \sum_{j=1}^{n} l_j \int_{\psi_{j-1}}^{\psi_j(\psi_m = T_2)} (1 - R\mu - RT_1 - Rt_2)\{-(a-bs)$$

$$\left(1 - \frac{\theta t_2^2}{2}\right)\left(t_2 + \frac{\theta t_2^3}{6}\right) + (p-(a-bs))\left(1 - \frac{\theta T_1^2}{2} - \frac{\theta t_2^2}{2}\right)\left(T_1 + \frac{\theta T_1^3}{6} + \mu\right)\Big\}dt_2 \Bigg]$$

$$= \frac{1}{T}\Bigg[h\{p-(a-bs)\}\left(\frac{\mu^2}{2} - \frac{R\mu^3}{3}\right) + \sum_{i=1}^{m} k_i\{p-(a-bs)\}\Big\{\left(\frac{1}{2} - R\mu\right)(\phi_i^2 - \phi_{i-1}^2)$$

$$- \frac{\theta}{12}(\phi_i^4 - \phi_{i-1}^4) + (\mu - R\mu^2)(\phi_i - \phi_{i-1}) + \left(-\frac{\theta\mu}{6} - \frac{R}{3}\right)(\phi_i^3 - \phi_{i-1}^3)\Big\}$$

(I, J)dt

$$-\sum_{j=1}^{n} l_j(a-bs)\left\{\left(\frac{1}{2}-\frac{R\mu}{2}-\frac{RT_1}{2}\right)\left(\psi_j^{\,2}-\psi_{j-1}^{\,2}\right)-\frac{R}{3}\left(\psi_j^{\,3}-\psi_{j-1}^{\,3}\right)-\frac{\theta}{12}\left(\psi_j^{\,4}-\psi_{j-1}^{\,4}\right)\right\}$$

$$+\sum_{j=1}^{n} l_j\{p-(a-bs)\}\left\{\left(T_1-\frac{\theta T_1^{\,3}}{3}+\mu-\frac{\mu\theta T_1^{\,2}}{2}-2R\mu T_1-RT_1^{\,2}-R\mu^2\right)\left(\psi_j-\psi_{j-1}\right)\right.$$

$$\left.-\left(\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)\left(\psi_j^{\,3}-\psi_{j-1}^{\,3}\right)-\left(\frac{RT_1}{2}+\frac{R\mu}{2}\right)\left(\psi_j^{\,2}-\psi_{j-1}^{\,2}\right)\right\}\Bigg] \qquad ...(15)$$

Deteriorated cost (DE) is given by

$$DE=\frac{C_3}{T}\left[\int_0^{T_1} e^{-R(\mu+t_1)}\theta t_1 I(t_1)dt_1+\int_0^{T_2} e^{-R(\mu+T_1+t_2)}\theta t_2 I(t_2)dt_2\right]$$

$$=\frac{C_3}{T}\left[\int_0^{T_1} e^{-R(\mu+t_1)}\theta t_1\{p-(a-bs)\}e^{-\theta t_1^{2}/2}\left(t_1+\frac{\theta t_1^{\,3}}{6}+\mu\right)dt_1+\int_0^{T_2} e^{-R(\mu+T_1+t_2)}\theta t_2\right.$$

$$\left.\left\{-(a-bs)e^{-\theta t_2^{2}/2}\left(t_2+\frac{\theta t_2^{\,3}}{6}\right)+(p-(a-bs))\left(T_1+\frac{\theta T_1^{\,3}}{6}+\mu\right)e^{-\theta\left(T_1^{2}+t_2^{2}\right)/2}\right\}dt_2\right]$$

$$=\frac{C_3}{T}\left[\int_0^{T_1}\left(1-R\mu-Rt_1-\frac{\theta t_1^{\,2}}{2}\right)\theta t_1\{p-(a-bs)\}\left(t_1+\frac{\theta t_1^{\,3}}{6}+\mu\right)dt_1+\right.$$

$$\int_0^{T_2}(1-R\mu-RT_1-Rt_2)\theta t_2\left\{-(a-bs)\left(1-\frac{\theta t_2^{\,2}}{2}\right)\left(t_2+\frac{\theta t_2^{\,3}}{6}\right)+(p-(a-bs))\right.$$

$$\left.\left.\left(T_1+\frac{\theta T_1^{\,3}}{6}+\mu\right)\left(1-\frac{\theta t_2^{\,2}}{2}-\frac{\theta T_1^{\,2}}{2}\right)\right\}dt_2\right]$$

$$=\frac{C_3}{T}\left[\{p-(a-bs)\}\left(\frac{\theta T_1^{\,3}}{3}+\frac{\mu\theta T_1^{\,2}}{2}+\frac{T_1\theta T_2^{\,2}}{2}+\frac{\mu\theta T_2^{\,2}}{2}\right)-(a-bs)\frac{\theta T_2^{\,3}}{3}\right] \qquad ..(16)$$

Shortage cost (SH) is given by

$$SH=\frac{C_4}{T}\int_0^{T_3} e^{-Rt_3}I_3(t_3)dt_3+\int_0^{T_4} e^{-R(\mu+T_1+T_2+T_3+t_4)}I_4(t_4)dt_4$$

$$=\frac{C_4}{T}\left[-\int_0^{T_3}(1-R\mu-RT_1-RT_2-Rt_3)(a-bs)t_3dt_3+\int_0^{T_4}(1-R\mu-RT_1-RT_2\right.$$

$$\left.-RT_3-Rt_4)\{p-(a-bs)\}(t_4-T_4)dt_4\right]$$

$$=\frac{C_4}{T}\{p-(a-bs)\}\left\{\frac{RT_4^{\,3}}{6}-\frac{T_4^{\,2}}{2}(1-R\mu-RT_1-RT_2-RT_3)\right\}-\frac{C_4}{T}(a-bs)\left\{\frac{T_3^{\,2}}{2}\right.$$

$$\left(1-R\mu-RT_1-RT_2-RT_3\right)-\frac{RT_3^{\,3}}{3}\Bigg\} \qquad ..(17)$$

The present value of the total relevant cost during the cycle is the sum of the ordering cost (OR), the holding cost (HD), the deteriorated cost (DE), the shortage cost (SH).

One has

TC (T_1, T_2, T_3, T_4, s)

= OR + HD + DE + SH

$$=\frac{1}{T}\Bigg[C_1+h\{p-(a-bs)\}\left(\frac{\mu^2}{2}-\frac{R\mu^3}{3}\right)+\sum_{i=1}^{m}k_i\{p-(a-bs)\}\{(\mu-R\mu^2)(\phi_i-\phi_{i-1})$$

$$+\left(\frac{1}{2}-R\mu\right)\left(\phi_i^{\,2}-\phi_{i-1}^{\,2}\right)-\left(\frac{\theta\mu}{6}+\frac{R}{3}\right)\left(\phi_i^{\,3}-\phi_{i-1}^{\,3}\right)-\frac{\theta}{12}\left(\phi_i^{\,4}-\phi_{i-1}^{\,4}\right)\Big\}-\sum_{j=1}^{n}l_j(a-bs)$$

$$\Big\{\left(\frac{1}{2}-\frac{R\mu}{2}\right)\left(\psi_j^2-\psi_{j-1}^2\right)-\frac{R}{3}\left(\psi_j^{\,3}-\psi_{j-1}^{\,3}\right)-\frac{\theta}{12}\left(\psi_j^{\,4}-\psi_{j-1}^{\,4}\right)\Big\}+\sum_{j=1}^{n}l_j\{p-(a-bs)\}\{(T_1$$

$$-\frac{\theta T_1^{\,3}}{3}+\mu-2R\mu T_1-RT_1^{\,2}-\frac{\mu\theta T_1^{\,2}}{2}-R\mu^2\Big)\left(\psi_j-\psi_{j-1}\right)-\left(\frac{RT_1}{2}+\frac{R\mu}{2}\right)\left(\psi_j^{\,2}-\psi_{j-1}^{\,2}\right)$$

$$-\left(\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)\left(\psi_j^3-\psi_{j-1}^3\right)\Big\}+C_3\{p-(a-bs)\}\left(\frac{\theta T_1^{\,3}}{3}+\frac{\theta\mu T_1^{\,2}}{2}+\frac{\theta T_1T_2^{\,2}}{2}+\frac{\theta\mu T_2^{\,2}}{2}\right)$$

$$-C_3(a-bs)\frac{\theta T_2^3}{3}-C_4(a-bs)\{(1-R\mu-RT_1-RT_2)\frac{T_3^{\,2}}{2}-\frac{RT_3^{\,3}}{3}\Big\}$$

$$+C_4\{p-(a-bs)\}\left\{\frac{RT_4^{\,3}}{6}-\left(1-R\mu-RT_1-RT_2-RT_3\right)\frac{T_4^{\,2}}{2}\right\} \qquad ..(18)$$

Now the first order necessary conditions for TC (T_1,T_2,T_3,T_4,s) to be minimum are

$$\frac{\partial TC}{\partial T_1}=0 \qquad ....(19)$$

$$\frac{\partial TC}{\partial T_2}=0 \qquad ...(20)$$

$$\frac{\partial TC}{\partial T_3}=0 \qquad ...(21)$$

$$\frac{\partial TC}{\partial T_4}=0 \qquad ...(22)$$

$$\frac{\partial TC}{\partial s}=0 \qquad ...(23)$$

Equation (19) gives

$$C_1+h\{p-(a-bs)\}\left(\frac{\mu^2}{2}-\frac{R\mu^3}{3}\right)+\sum_{i=1}^{m}k_i\{p-(a-bs)\}\{(\mu-R\mu^2)(\phi_i-\phi_{i-1})$$

$$+\left(\frac{1}{2}-R\mu\right)(\phi_i^2-\phi_{i-1}^2)-\left(\frac{\theta\mu}{6}+\frac{R}{3}\right)(\phi_i^3-\phi_{i-1}^3)-\frac{\theta}{12}(\phi_i^4-\phi_{i-1}^4)\}-\sum_{j=1}^{n}l_j(a-bs)$$

$$\left\{\left(\frac{1}{2}-R\mu\right)(\psi_j^2-\psi_{j-1}^2)-\frac{R}{3}(\psi_j^3-\psi_{j-1}^3)-\frac{\theta}{12}(\psi_j^4-\psi_{j-1}^4)\right\}-\sum_{j=1}^{n}l_j\{p-(a-bs)\}$$

$$\{(\psi_j-\psi_{j-1})(1-\theta T_1^2-\mu\theta T_1-2R\mu-2RT_1-T_1+\frac{\theta T_1^3}{3}-\mu+\frac{\mu\theta T_1^2}{2}+2R\mu T_1$$

$$+RT_1^2+R\mu^2)+(\psi_j^2-\psi_{j-1}^2)\left(-\frac{R}{2}+\frac{RT_1}{2}+\frac{R\mu}{2}\right)+(\psi_j^3-\psi_{j-1}^3)\left(-\frac{\theta}{6}+\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)\}$$

$$-C_3\{p-(a-bs)\}\left(\theta T_1^2+\theta\mu T_1-\frac{\theta T_1^3}{3}-\frac{\theta\mu T_1^2}{2}+\frac{\theta T_2^2}{2}-\frac{\theta T_1T_2^2}{2}-\frac{\theta\mu T_2^2}{2}\right)$$

$$-C_3(a-bs)\frac{\theta T_2^3}{3}-C_4(a-bs)\left\{\frac{RT_3^2}{2}-\frac{RT_3^3}{3}+(1-R\mu-RT_1-RT_2)\frac{T_3^2}{2}\right\}$$

$$-C_4\{p-(a-bs)\}\left\{\frac{T_4^2R}{2}+\frac{T_4^2}{2}(1-R\mu-RT_1-RT_2-RT_3)-\frac{RT_4^3}{6}\right\}=0 \qquad ..(24)$$

Equation (20) gives

$$C_1+h\{p-(a-bs)\}\left(\frac{\mu^2}{2}-\frac{R\mu^3}{3}\right)+\sum_{i=1}^{m}k_i\{p-(a-bs)\}\{(\mu-R\mu^2)(\phi_i-\phi_{i-1})$$

$$+\left(\frac{1}{2}-R\mu\right)(\phi_i^2-\phi_{i-1}^2)-\left(\frac{\theta\mu}{6}+\frac{R}{3}\right)(\phi_i^3-\phi_{i-1}^3)-\frac{\theta}{12}(\phi_i^4-\phi_{i-1}^4)\}-\sum_{j=1}^{n}l_j(a-bs)$$

$$(\psi_j^2-\psi_{j-1}^2)-\frac{R}{3}(\psi_j^3-\psi_{j-1}^3)-\frac{\theta}{12}(\psi_j^4-\psi_{j-1}^4)\}+\sum_{j=1}^{n}l_j\{p-(a-bs)\}\left\{\left(T_1-\frac{\theta T_1^3}{3}+\mu\right.\right.$$

$$-\frac{\mu\theta T_1^2}{2}-2R\mu T_1-RT_1^2-R\mu^2)(\psi_j-\psi_{j-1})-\left(\frac{RT_1}{2}+\frac{R\mu}{2}\right)(\psi_j^2-\psi_{j-1}^2)-\left(\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)$$

$$(\psi_j^3-\psi_{j-1}^3)\}+C_3\{p-(a-bs)\}\left(\frac{\theta T_1^3}{3}+\frac{\theta\mu T_1^2}{2}-\theta T_1T_2-\theta\mu T_2-\frac{\theta T_1T_2^2}{2}-\frac{\theta\mu T_2^2}{2}\right)$$

$$-C_3(a-bs)\left(\frac{\theta T_2^3}{3}-\theta T_2^2\right)-C_4(a-bs)\left\{(1-R\mu-RT_1-RT_2)\frac{T_3^2}{2}-\frac{RT_3^3}{3}+\frac{RT_2^2}{2}\right\}$$

$$-C_4\{p-(a-bs)\}\left\{\frac{RT_4^2}{2}+(1-R\mu-RT_1-RT_2-RT_3)\frac{T_4^2}{2}-\frac{RT_4^3}{6}\right\}=0 \qquad ..(25)$$

Equation (21) gives

$$C_1 + h\{p-(a-bs)\}\left(\frac{\mu^2}{2}-\frac{R\mu^3}{3}\right)+\sum_{i=1}^{m} k_i\{p-(a-bs)\}\{(\mu-R\mu^2)(\phi_i-\phi_{i-1})$$

$$+\left(\frac{1}{2}-R\mu\right)(\phi_i^2-\phi_{i-1}^2)-\left(\frac{\theta\mu}{6}+\frac{R}{3}\right)(\phi_i^3-\phi_{i-1}^3)-\frac{\theta}{12}(\phi_i^4-\phi_{i-1}^4)\Big\}-\sum_{j=1}^{n} l_j(a-bs)$$

$$(\psi_j^2-\psi_{j-1}^2)-\frac{R}{3}(\psi_j^3-\psi_{j-1}^3)-\frac{\theta}{12}(\psi_j^4-\psi_{j-1}^4)\Big\}+\sum_{j=1}^{n} l_j\{p-a+bs\}\left\{\left(T_1-\frac{\theta T_1^3}{3}+\mu\right.\right.$$

$$-\frac{\mu\theta T_1^2}{2}-2R\mu T_1-RT_1^2-R\mu^2\Big)(\psi_j-\psi_{j-1})-\left(\frac{RT_1}{2}+\frac{R\mu}{2}\right)(\psi_j^2-\psi_{j-1}^2)-\left(\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)$$

$$(\psi_j^3-\psi_{j-1}^3)\Big\}+C_3\{p-(a-bs)\}\left(\frac{\theta T_1^3}{3}+\frac{\theta\mu T_2^2}{2}+\frac{\theta T_1 T_2^2}{2}+\frac{\theta\mu T_2^2}{2}\right)-C_3(a-bs)\frac{\theta T_2^3}{3}$$

$$-C_4(a-bs)\left\{(1-R\mu-RT_1-RT_2)\frac{T_3^2}{2}-(1-R\mu-RT_1-RT_2)T_3-RT_3^2-\frac{RT_3^3}{3}\right\}$$

$$-C_4\{p-(a-bs)\}\left\{\frac{T_4^2}{2}(1-R\mu-RT_1-RT_2-RT_3)-\frac{RT_4^3}{6}+\frac{RT_4^2}{2}\right\}=0 \qquad ..(26)$$

Equation (22) gives

$$C_1 + h\{p-(a-bs)\}\left(\frac{\mu^2}{2}-\frac{R\mu^3}{3}\right)+\sum_{i=1}^{m} k_i\{p-(a-bs)\}\{(\mu-R\mu^2)(\phi_i-\phi_{i-1})$$

$$+\left(\frac{1}{2}-R\mu\right)(\phi_i^2-\phi_{i-1}^2)-\left(\frac{\theta\mu}{6}+\frac{R}{3}\right)(\phi_i^3-\phi_{i-1}^3)-\frac{\theta}{12}(\phi_i^4-\phi_{i-1}^4)\Big\}-\sum_{j=1}^{n} l_j(a-bs)$$

$$\left\{\left(\frac{1}{2}-R\mu\right)(\psi_j^2-\psi_{j-1}^2)-\frac{R}{3}(\psi_j^3-\psi_{j-1}^3)-(\psi_j^4-\psi_{j-1}^4)\frac{\theta}{12}\right\}+\sum_{j=1}^{n} l_j\{p-(a-bs)\}\{(T_1$$

$$-\frac{\theta T_1^3}{3}+\mu-\frac{\mu\theta T_1^2}{2}-2R\mu T_1-RT_1^2-R\mu^2\Big)(\psi_j-\psi_{j-1})-\left(\frac{RT_1}{2}+\frac{R\mu}{2}\right)(\psi_j^2-\psi_{j-1}^2)$$

$$-\left(\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)(\psi_j^3-\psi_{j-1}^3)\Big\}+C_3\{p-(a-bs)\}\left(\frac{\theta T_1^3}{3}+\frac{\theta\mu T_1^2}{2}+\frac{\theta T_1 T_2^2}{2}+\frac{\theta\mu T_2^2}{2}\right)$$

$$\left\{(1-R\mu-RT_1-RT_2-RT_3)\left(\frac{T_4^2}{2}-T_4\right)+\frac{RT_4^2}{2}-\frac{RT_4^3}{6}\right\}=0 \qquad ..(27)$$

Equation (23) gives

$$C_1 + h\left(\frac{\mu^2}{2} - \frac{R\mu^3}{3}\right)(p-a+bs+b) + \sum_{j=1}^{n} k_j (p-a+bs-b)\Big\{(\mu - R\mu^2)(\phi_j - \phi_{j-1})$$

$$+\left(\frac{1}{2} - R\mu\right)(\phi_j^2 - \phi_{j-1}^2) - \left(\frac{\theta\mu}{6}+\frac{R}{3}\right)(\phi_j^3-\phi_{j-1}^3) - \frac{\theta}{12}(\phi_j^4-\phi_{j-1}^4)\Big\} + \sum_{j=1}^{m} l_j(a-bs)$$

$$(\psi_j^2-\psi_{j-1}^2) - \frac{R}{3}(\psi_j^3-\psi_{j-1}^3) - \frac{\theta}{12}(\psi_j^4 - \psi_{j-1}^4)\Big\} + \sum_{j=1}^{m} l_j(p-a+bs-b)\Big\{(T_1$$

$$-\frac{\theta T_1^3}{3} + \mu - 2R\mu T_1 - RT_1^2 - \frac{\mu\theta T_1^2}{2} - R\mu^2\Big)(\psi_j - \psi_{j-1}) - \left(\frac{RT_1}{2}+\frac{R\mu}{2}\right)(\psi_j^2-\psi_{j-1}^2)$$

$$-\left(\frac{\theta T_1}{6}+\frac{\mu\theta}{6}\right)(\psi_j^3-\psi_{j-1}^3)\Big\} + C_3(p-a+bs-b)\left(\frac{\theta T_1^3}{3}+\frac{\theta\mu T_1^2}{2}+\frac{\theta T_1 T_2^2}{2}+\frac{\theta\mu T_2^2}{2}\right)$$

$$-C_3(a-bs)\frac{\theta T_2^3}{3} - C_4(a-bs+b)\left\{(1-R\mu-RT_1-RT_2)\frac{T_3^2}{2} - \frac{RT_3^3}{3}\right\}$$

$$+C_4(p-a+bs-b)\left\{\frac{RT_4^3}{6} - (1-R\mu-RT_1-RT_2-RT_3)\frac{T_4^2}{2}\right\} = 0 \qquad (28)$$

Using software Matlab 7.0, the equations (24), (25), (26), (27) & (28) are solved and by substituting the values of T_1, T_2, T_3, T_4 & s in equation (23), TC is obtained.

NUMERICAL ILLUSTRATIONS:

In this section, a numerical example is tested to show accuracy of the model and the solution procedure.

The parameters' quantities are:

$a = 50$, $b = .9$, C_1 = Rs. 200, h = Rs. 3, k_i = Rs. (h + i), $l_j = (h + \sum_{i=1}^{m} k_i + j)$,

C_2 = Rs. 10, C_4 = Rs. 12, $\theta = 0.02$ unit,

$\mu = 0.4$, R = 0.1 unit.

Optimal solution is

$T_1 = 1.75$, $T_2 = 2.04$, $T_3 = 0.89$, $T_4 = 0.38$,

TC = Rs.4239.76

Outsourced quantity per order = 1932.78 units.

SENSITIVTY ANALYSIS:

We will now study the sensitivity of the optimal solution to changes in the values of the different parameters associated with the inventory system in Example 1. The results are shown below. The percentage cost increase (PCI) is

$$PCI = \frac{TC - TC^*}{TC^*} \times 100\%$$

Table 1. Optimal solution when the parameter θ is changed by 10%

θ	T_1	T_2	T_3	T_4	s	TC	PCI
0.024	1.74458	2.04000	0.89090	0.37922	31.65098	4239.930	.004
0.022	1.75006	2.04013	0.89312	0.38017	30.99186	4239.874	.002
0.018	1.75864	2.04621	0.89948	0.38647	30.65336	4239.659	-.002
0.016	1.75993	2.04976	0.90099	0.38857	30.12665	4239.600	-.003

The following points are noted from above table:

- Inventory period T_1, T_2, T_3 and T_4 decreases as deterioration rate θ increases.
- Selling price increases as the deterioration rate θ increases.
- The value of PCI increases as the deterioration rate θ increases.

Table 2. Optimal solution when the parameter μ is changed by 10%

μ	T_1	T_2	T_3	T_4	s	TC	PCI
.48	1.77879	2.05121	0.89665	0.38976	30.16323	4239.384	-0.008
.44	1.76212	2.04989	0.89488	0.38587	30.65449	4239.538	-0.005
.36	1.74804	2.03437	0.89264	0.38223	30.97098	4239.896	0.003
.32	1.74108	2.02997	0.89034	0.38088	31.34112	4240.013	0.005

The following points are noted from above table:

- We notice that inventory period T_1, T_2, T_3, and T_4 increases as life time of items μ increases.
- The selling price s decreases as life time of items μ increases.
- The value of PCI decreases as life time of items μ increases.

Table 3. Optimal solution when the parameter R is changed by 10%

R	T_1	T_2	T_3	T_4	s	TC	PCI
0.12	1.74899	2.03822	0.88901	0.37444	30.96332	4240.455	0.016
0.11	1.75100	2.04001	0.89006	0.37991	30.91026	4239.984	0.005
0.09	1.76295	2.04915	0.89752	0.38878	30.23981	4239.568	-0.004
0.08	1.76666	2.05117	0.89992	0.39001	30.11098	4239.245	-0.012

The following points are noted from above table:

- We notice that inventory period T_1, T_2, T_3 and T_4 decreases as inflation parameter R increases.
- Selling price increases as inflation rate R increases.
- The value PCI increases as inflation rate R increases.

CONCLUSIONS

In this chapter, a supply chain inventory model is developed under the realistic condition incremental holding cost. In this model, demand is taken price dependent. The item is deteriorating after life time of the item. Shortage and inflation is also taken in account. Sensitivity analyses of some parameter also discuss.

REFERENCES

1. Alfares, H. K. (2007): Inventory model with stock level dependent demand rate and variable holding cost. International journal of production Economics, 108(1-2), 259-265.
2. Balkhi, Z.T. and Benkherouf, L. (1996): A production lot size inventory model for deteriorating items and arbitrary production and demand rate. European Journal of Operational Research, 92, 302-309.
3. Bose, S., Goswami, A. and Chaudhari, K. S. (1995): An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting, Journal of the Operational Research Society, 46(6), 771-782.
4. Burwell, T. H., Dave, D. S., Fitzpatrick, K. E. and Roy, M. R. 1997: Economic lot size model for price-dependent demand under quantity and freight discounts, International Journal of Production Economics, 48(2), 141-155.
5. Buzacott, J. (1975): Economic order quantities with inflation. Operations Research Quarterly, 26(3), 553-558.
6. Giri, B. C., Goswami, A. and Chaudhuri, K. S. (1996): An EOQ model for deteriorating items with time varying demand and costs. Journal of the Operational Research Society, 47(11), 1398-1405.
7. Hwang H. and Hahn K. H. (2000): An optimal procurement policy for items with an inventory level

dependent demand rate and fixed life time. European Journal of Operation Research, 127(3), 537-545.

8. Mishra, R. B. (1979): A note on optimal inventory management under inflation. Naval Research Logistics Quarterly, 26(2), 161-165.

9. Mondal, B., Bhunia, A. K., Maiti, M. 2003: An inventory system of ameliorating items for price dependent demand rate, Computers and Industrial Engineering, 45(3), 443-456.

10. Padmanabhan, G. and Vrat, P. (1995): EOQ models for perishable items under stock dependent selling rate. European Journal of Operational Research, 86, 281-292.

11. Roychowdhury, M. and Chaudhuri, K.S. (1983): An order level inventory model for deteriorating items with finite rate of replenishment. Opsearch, 20, 99-106.

12. Yang, H. L., Teng, J. T. and Chern, M. S. (2001): Deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuation demand. Naval Research Logistics, 48(2), 144-158.

13. Yang, H. L. (2004): Two-warehouse inventory models for deteriorating items with shortages under inflation. European Journal of Operational Research, 157(2), 344-356.

14. Yang, H. L. (2005): A comparison among various partial backlogging inventory lot-size models for deteriorating items on the basis of maximum profit. International Journal of Production Economics, 96, 119-128.

15. You, S.P. (2005): Inventory policy for products with price and time-dependent demands, Journal of the Operational Research Society, 56, 870-873.